Signatures of the topological spin of Josephson vortices in topological superconductors

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Realization of non-abelian quasi-particles known as Majorana fermions is an ongoing challenge for physicists exploring topological states of matter. Towards achieving this goal, we recently suggested that Josephson vortices in topological josephson junctions (TJJ) would constitute such Majorana fermions and retain the exchange statistics of bulk vortices. Here we corroborate this hypothesis by finding the universal exchange phase of Josephson vortices and suggesting an experiment to measure it by.

The setup

Our setup consists of a josephson vortex trapped in an annular topological superconduting junction, which encloses both an electric charge and magnetic flux. The vortex is driven into a persistent motion through an Aharonov-Casher effect[1, 2].

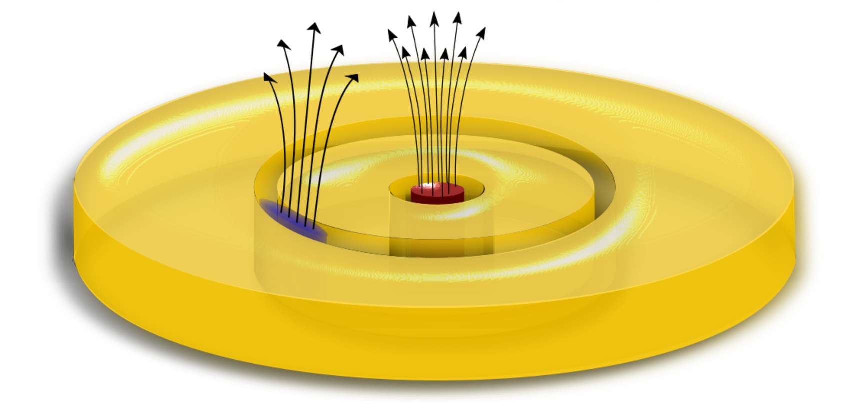

Figure 1: An annular TJJ trapping a single soliton. The soliton is depicted in blue. Counter propagating Majorana edge states are nucleated in the junction. A charge Q and phase Φ are induced externally within the central region (red).

Effective Hamiltonian and Topological Spin

The dynamics of a TJJ is governed by a modified Sine-Gordon Hamiltonian, where the regular bosonic degrees of freedom couple with the low lying Majorana fermions[3, 4]. The Hamiltonian in terms of particlelike coordinates, $[p, q] = -i\hbar$, associated with the soliton[5] is

$$H_s = \frac{1}{2m_s}\left[p - \frac{2\pi}{L}\left(\mathcal{L} - \bar{\mathcal{L}}\right)\right]^2 + \frac{2\pi v}{L}(\mathcal{L}+\bar{\mathcal{L}}) + i\int \mathrm{d}x\, W(x,q)\psi(x)\bar{\psi}(x), \quad (1)$$

- $\psi(x)$ $(\bar{\psi}(x))$ is a Majorana operator for the external (internal) edge.
- $W(x,q) = m(q)\cos\left[\varphi(x-q)/2\right]$ is the Majorana mass term.
- $\mathcal{L} \equiv \sum_{n>0} n\psi_{-n}\psi_n + \mathcal{L}_0(N_v)$, $\quad \bar{\mathcal{L}} \equiv \sum_{n>0} n\bar{\psi}_{-n}\bar{\psi}_n + \bar{\mathcal{L}}_0(\bar{N}_v)$,
- $\mathcal{L}_0$ $(\bar{\mathcal{L}}_0)$ defined as the ground state contribution
- N_v $(\bar{N}_v)$ denotes the number of vortices enclosed by the external (internal) edge.
- $n \in \mathbb{Z}$ for an odd number of vortices enclosed by the edge, otherwise $n \in \mathbb{Z} + 1/2$.

We have calculated the ground state contribution to the vector potential and found that it is $2\pi/L$ times the topological spin,

$$\frac{2\pi}{L}(\mathcal{L}_0 - \bar{\mathcal{L}}_0) = (-1)^{N_v}\frac{2\pi}{L}\frac{1}{16}. \quad (2)$$

Numerical calculation of the Topological Spin

For short Josephson junction, the Berry phase that the ground state $|\Omega_q\rangle$ accumulates as function of the soliton's position, q was calculated.

- The Berry connection for systems that posses particle-hole symmetry[6] is given by

$$i\langle\Omega_q|\partial_q\Omega_q\rangle = \frac{i}{4}\mathrm{Tr}\left[(1+gg^\dagger)^{-1}\left(g'g^\dagger - g(g^\dagger)'\right)\right], \quad (3)$$

  where $g = (VU^{-1})^*$ and the columns of $(U^T\ V^T)$ are eigenstates that correspond to positive energy values in an acceding order.
- We use the momentum states in a truncated Hilbert space.
- The soliton translation operator is identified which enables us to perform the derivative symbolically.
- We reproduced the result of Eq.2 to machine precision.

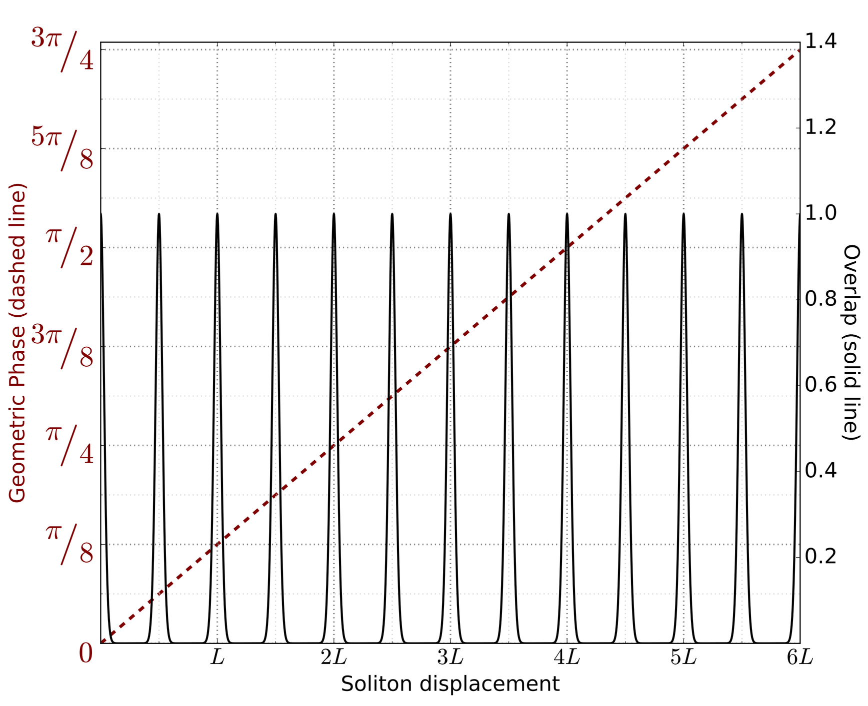

Figure 2: Brown line describes the accumulated Berry phase by each persisting soliton in the presence of a vortex within the central region. In addition, Black line describes the overlap norm of two counter-propagating solitons, which becomes non zero at half cycles. At these points the phase of each soliton acquires its universal values $n\pi/16$, $n \in \mathbb{Z}$.

Regular vs. Topological Josephson junctions

- The persistent motion of the topological Josephson vortex can be manipulated by two knobs - an externally induced charge and a magnetic flux, both enclosed in the central area of the annular TJJ.
- In contrast, a non-topological Josephson vortex remains unaffected by coherent fermion parity changing effects.

The Josephson vortex Energy spectrum

The energy spectrum of the Josephson vortex in the presence of an externally induced Aharonov-Casher charge Q within the central region is

$$E_s = E_c\left[\frac{Q}{2e} + \left(\frac{n_f}{4} + \frac{n_v}{16}\right) - N_b\right]^2, \quad (4)$$

- E_c is the charging energy for the junction,.
- $n_f = (-1)^{N_f}$ is the fermion parity within the junction centeral region (N_f is the fermion number).
- $n_v = (-1)^{N_v}$ is the parity of the number of vortices within the same region.
- $N_b \in \mathbb{Z}$ is the relative number of Cooper pairs between the two superconducting plates.

Both the velocity of the persisting soliton and the voltage across the junction are proportional to the gradient of the energy, $V, v_s \propto \partial_Q E$.

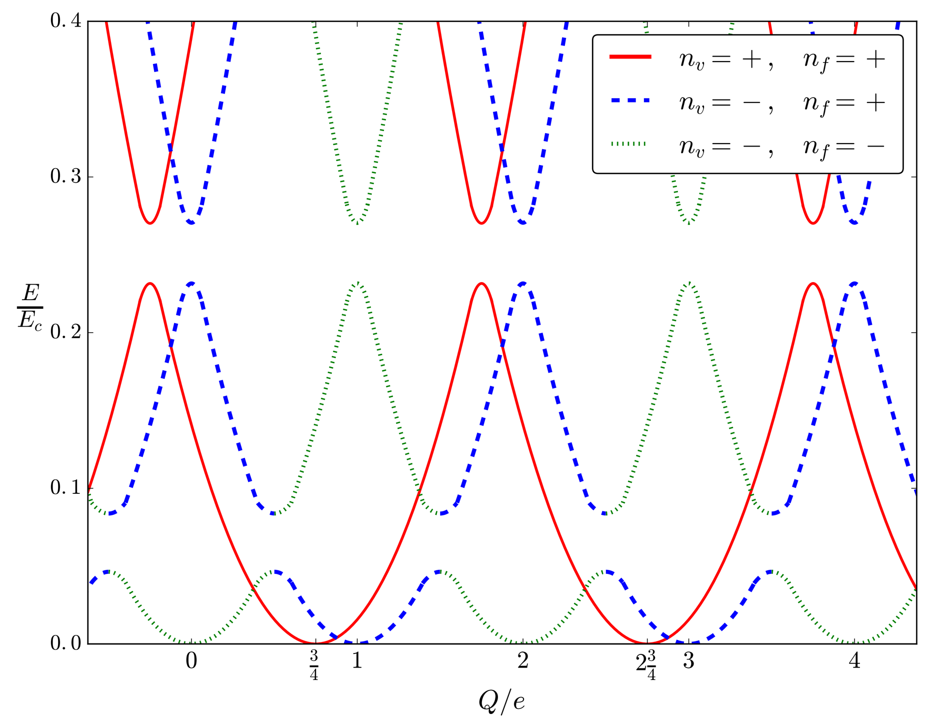

Figure 3: Red describe the soliton energy in the presence of an even number of vortices. Blue and green describe the case with an odd number of vortices (for even and odd fermion parities respectively). Fermion parity changing effects open a gap between the green and blue, disorder opens a gap between the same color.

Conclusions

- The universal phase depends only on the parity of the number of vortices enclosed by the junction. This phase is $\pm 2\pi$ times the topological spin of the Josephson vortex and is proportional to the Chern number[7].
- The topological spin can be measured through its effect on the junction's voltage characteristics.
- Our platform and the topological spin, in particular, can be exploited to form the sought-after, $\pi/8$ magic phase gate. This gate is necessary to complete a set of universal quantum gates.

Acknowledgements

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